

# Math 115

## Spring 2017

### Lecture 29

John works 3 times as fast as Mary.  
 Together, they can do a certain job in 21 hrs.  
 How long does it take each one to do the  
 job alone?

John	Mary	Rate for John	Mary
1 hr.	3 hrs	$\frac{1}{x}$	$\frac{1}{3x}$
2 hrs.	6 hrs.		
$x$ hrs	$3x$ hrs		

$$\frac{1}{x} \cdot 21 + \frac{1}{3x} \cdot 21 = 1$$

$$\frac{21}{x} + \frac{7}{x} = 1 \quad \text{LCD} = x$$

$$21 + 7 = x$$

$$\boxed{x = 28}$$

John  $\rightarrow$  28 hrs

Mary  $\rightarrow$  84 hrs

John takes 3 hrs longer than Mary to do certain job. Together, they can do the same job in 2 hrs.

How long if they work alone?

John	Mary
4 hrs.	1 hr.
5 hrs	2 hrs.
$(x+3)$ hrs	$x$ hrs.

$$\underbrace{\frac{1}{x} \cdot 2}_{\text{Rate} \cdot \text{time}} + \underbrace{\frac{1}{x+3} \cdot 2}_{\text{Rate} \cdot \text{time}} = 1$$

1 Complete Job

$$\boxed{\frac{2}{x}} + \boxed{\frac{2}{x+3}} = \boxed{1}$$

$$\text{LCD} = x(x+3)$$

$$2x + 6 + 2x = x^2 + 3x$$

$$x^2 + 3x - 4x - 6 = 0$$

$$x^2 - x - 6 = 0$$

$$2(x+3) + 2x = x(x+3)$$

$$x^2 - x - 6 = 0$$

$a=1$   $b=-1$   $c=-6$

$$b^2 - 4ac =$$

$$(-1)^2 - 4(1)(-6) = 1 + 24 = 25$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{25}}{2(1)}$$

$$= \frac{1 \pm 5}{2} \rightarrow x = \frac{1+5}{2} = 3$$

~~$$x = \frac{1-5}{2} = -2$$~~

Mary can do the job alone in 3 hrs,  
and John can do it in 6 hrs

John drove 130 miles in the same time that Mary drove 140 miles.

John was driving 5 mph slower than Mary.

①

	$r \cdot t = d$		
John	$x-5$	$t$	130
Mary	$x$	$t$	140

Mary 70 mph  
John 65 mph.

② Find speed for both.

$$\frac{130}{x-5} = \frac{140}{x}$$

$$14(x-5) = 13x$$

$$14x - 70 = 13x$$

$$\boxed{x=70}$$

Mary drove 90 miles in the City and 130 miles on the highway. ①

Total time 4 hrs.

She drove 20 mph faster on the highway than in the City. ② Set-up the eqn, and solve.

	$r$	$t$	$=d$
City	$x$	$t_1$	$=90$
Hwy	$x+20$	$t_2$	$=130$

$$t_1 + t_2 = 4$$

$$\frac{90}{x} + \frac{130}{x+20} = 4 = \frac{4}{1}$$

Divide by 2 to reduce

$$\frac{45}{x} + \frac{65}{x+20} = 2$$

$$LCD = x(x+20)$$

$$45(x+20) + 65x = 2x \cdot (x+20)$$

$$45(x+20) + 65x = 2x \cdot (x+20)$$

$$45x + 900 + 65x = 2x^2 + 40x$$

$$2x^2 + 40x - 110x - 900 = 0$$

$$2x^2 - 70x - 900 = 0$$

Divide by 2

$$x^2 - 35x - 450 = 0$$

45 mph → City

65 mph → Hwy

$$\rightarrow a=1, b=-35, c=-450$$

$$b^2 - 4ac = (-35)^2 - 4(1)(-450) \\ = 3025$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-35) \pm \sqrt{3025}}{2(1)}$$

$$= \frac{35 \pm 55}{2}$$

$$x = \frac{35+55}{2} = 45 \quad x = \frac{35-55}{2}$$

If  $x^2 = K$ ,  $K \geq 0$ , then  $x = \pm \sqrt{K}$

Square-Root Method

Solve  $x^2 = 100$

$$x = \pm \sqrt{100}$$

$$\boxed{x = \pm 10} \quad \{\pm 10\}$$

Solve  $(x+4)^2 = 49$

$$x+4 = \pm \sqrt{49}$$

$$x+4 = \pm 7$$

$$\rightarrow x = -4 \pm 7$$

$$x = -4 + 7 = 3$$

$$x = -4 - 7 = -11$$

$$\{3, -11\}$$

use Square - Root method to Solve

$$\textcircled{1} x^2 = 64$$

$$x = \pm \sqrt{64}$$

$$\boxed{x = \pm 8} \quad \{\pm 8\}$$

$$\textcircled{2} (x-3)^2 = 81$$

$$x-3 = \pm \sqrt{81}$$

$$x-3 = \pm 9 \quad \rightarrow x = 3+9 = 12$$

$$x = 3 \pm 9 \quad \rightarrow x = 3-9 = -6$$

$$\{-6, 12\}$$

$$\textcircled{3} (2x-5)^2 = 121$$

$$2x-5 = \pm \sqrt{121}$$

$$2x = 5 \pm 11 \quad \rightarrow x = \frac{5+11}{2} = 8$$

$$x = \frac{5 \pm 11}{2}$$

$$x = \frac{5-11}{2} = -3$$

$$\Rightarrow \{-3, 8\}$$

Making a perfect - Sqr:

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

$$x^2 + 8x + 16 = \left(x + 4\right)^2$$

$$x^2 - 10x + 25 = \left(x - 5\right)^2$$

$$x^2 + 7x + \frac{49}{4} = \left(x + \frac{7}{2}\right)^2$$

$$x^2 - 15x + \frac{225}{4} = \left(x - \frac{15}{2}\right)^2$$

$$x^2 + \frac{1}{5}x + \frac{1}{100} = \left(x + \frac{1}{10}\right)^2$$

Red arrows indicate the calculation of the constant term:  $\frac{1}{2} \cdot \frac{1}{5} = \frac{1}{10}$ , which is then squared to get  $\frac{1}{100}$ .

$$x^2 - \frac{2}{3}x + \frac{1}{9} = \left(x - \frac{1}{3}\right)^2$$

Red arrows indicate the calculation of the constant term:  $\frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$ , which is then squared to get  $\frac{1}{9}$ .

$$x^2 + \frac{3}{4}x + \frac{9}{64} = \left(x + \frac{3}{8}\right)^2$$

Red arrows indicate the calculation of the constant term:  $\frac{3}{4} \cdot \frac{1}{2} = \frac{3}{8}$ , which is then squared to get  $\frac{9}{64}$ .

Solving  $ax^2 + bx + c = 0$ ;  $a \neq 0$  by

Completing the square:

$$x^2 + 6x + 5 = 0$$

① Move the constant to the other side

$$x^2 + 6x = -5$$

② Make a perfect-Square on the left-hand side.

$$x^2 + 6x + 9 = -5 + 9$$

$$(x + 3)^2 = 4$$

③ Now, use the sq-root method to Solve

$$(x+3)^2 = 4$$

$$x+3 = \pm\sqrt{4}$$

$$x+3 = \pm 2$$

$$x = -3 \pm 2$$

$$\rightarrow x = -3 + 2 = -1$$

$$x = -3 - 2 = -5$$

$$\{-1, -5\}$$

Solve  $x^2 - 8x - 9 = 0$  by Completing the Square method.

$$x^2 - 8x + 16 = 9 + 16$$

$$(x-4)^2 = 25$$

Now use S.R.M.

$$x-4 = \pm\sqrt{25}$$

$$x = 4 \pm 5$$

$$\boxed{x=9}$$

$$\boxed{x=-1}$$

$$\{-1, 9\}$$

Solve by Completing the sqr method:

$$x^2 - 3x - 10 = 0$$

$$x^2 - 3x + \frac{9}{4} = \frac{10 \cdot 4}{4} + \frac{9}{4}$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{49}{4}$$

by S.R.M.

$$x - \frac{3}{2} = \pm \frac{\sqrt{49}}{\sqrt{4}} \Rightarrow x = \frac{3}{2} \pm \frac{7}{2}$$

$$x = \frac{3}{2} + \frac{7}{2} = \frac{10}{2} = \boxed{5}, \quad x = \frac{3}{2} - \frac{7}{2} = \frac{-4}{2} = \boxed{-2} \quad \{-2, 5\}$$

Final Exam:

Solve by Quadratic Formula:

$$(2x + 1)(2x - 3) = 5$$

$$4x^2 - 6x + 2x - 3 - 5 = 0$$

$$4x^2 - 4x - 8 = 0$$

Divide by 4

$$x^2 - x - 2 = 0$$

$$a=1, b=-1, c=-2$$

$$b^2 - 4ac = (-1)^2 - 4(1)(-2)$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{9}}{2} = \frac{1 \pm 3}{2}$$

$$\boxed{x=2} \quad \boxed{x=-1}$$

Hint: FOIL,  
Simplify, write  
in  
 $ax^2 + bx + c = 0$   
form.



Find eqn of a line that contains  $(-3, 2)$  and is perpendicular to the line  $2x - 5y = 10$ .

$$m = -\frac{5}{2}$$

Use point-slope form

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{5}{2}(x - 3)$$

$$y - 2 = -\frac{5}{2}(x + 3)$$

multiply by LCD=2

$$-5y = -2x + 10$$

$$y = \frac{2}{5}x - 2$$

$$m = \frac{2}{5}$$

$$2y - 4 = -5(x + 3)$$

$$2y - 4 = -5x - 15$$

$$2y = -5x - 11$$

Divide by 2

$$y = -\frac{5}{2}x - \frac{11}{2}$$

① Review Exams 1, 2, and 3.

② Review Recent materials on rational expressions.

③ Review word problems

④ Review Factoring, Exponential rules, operations with polynomials.